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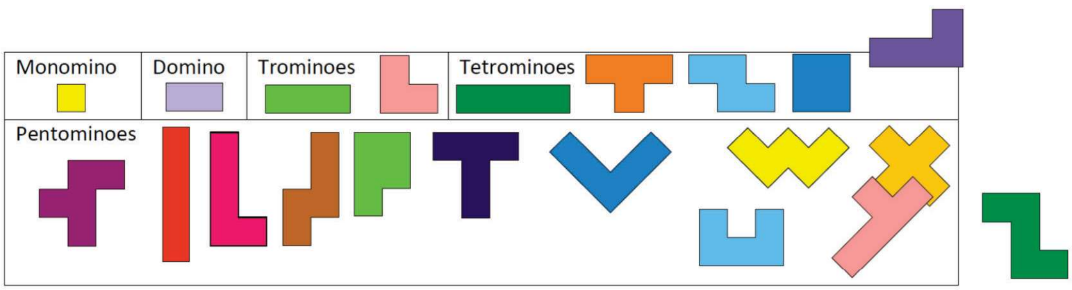
'POLYOMINOES' – REVISITED

In our first article, *Investigations with Pentominoes,* in the **Low Floor High Ceiling** series in **At Right Angles**, Vol 4, Issue 1, we proved by induction that every polyomino has an even number of sides. It was a long visual proof considering various cases and subcases. Here is a shorter proof of the same result, sparked by discussions with the B. Math students at Indian Statistical Institute.



Any polyomino is made of squares of the same size. So each internal angle of a polyomino must be an odd multiple of a right angle, i.e., either 90° or 270°. This means that we can always arrange any polyomino on the Cartesian plane so that its sides are parallel to the axes. We will refer to the sides parallel to the x-axis as 'horizontal' and those parallel to the y-axis as 'vertical.' Now horizontal and vertical sides must alternate, i.e., between any two consecutive horizontal sides there must be a vertical side, and vice versa. Since polyominoes are polygons with closed curves, the number of horizontal sides must equal the number of the vertical sides. Therefore, if any polyomino has k horizontal sides, then it must have k vertical sides as well, making it a $2k$-gon, i.e., a polygon with even number of sides.

The same argument extends to polyominoes with holes. A n-omino with a hole can be thought as a $(n + m)$-omino from which a m-omino has been taken out, for some $m < n$, where the $(n + m)$-omino and the m-omino are without any holes and have $2k$ and $2l$ number of sides respectively. So the number of sides for this n-omino is the sum of the number of sides of the $(n + m)$-omino and that of the m-omino, i.e., the number of sides for the n-omino with a hole $= 2k + 2l = 2(k + l)$, which is an even number. A similar argument can be extended for a polyomino with multiple holes.

– Swati Sircar